Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2012

(Junior Section, Round 2)

Saturday, 23 June 2012 **0930-1230**

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1. Let O be the centre of a parallelogram $ABCD$ and P be any point in the plane. Let M, N be the midpoints of AP, BP, respectively and Q be the intersection of MC and ND. Prove that O, P and Q are collinear.

2. Does there exist an integer A such that each of the ten digits $0, 1, \ldots, 9$ appears exactly once as a digit in exactly one of the numbers A, A^2, A^3.

3. In $\triangle ABC$, the external bisectors of $\angle A$ and $\angle B$ meet at a point D. Prove that the circumcentre of $\triangle ABD$ and the points C, D lie on the same straight line.

4. Determine the values of the positive integer n for which the following system of equations has a solution in positive integers $x_1, x_2, \ldots, x_n$. Find all solutions for each such n.

$$x_1 + x_2 + \cdots + x_n = 16 \qquad (1)$$

$$\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n} = 1 \qquad (2)$$

5. Suppose $S = a_1, a_2, \ldots, a_{15}$ is a set of 15 distinct positive integers chosen from $2, 3, \ldots, 2012$ such that every two of them are coprime. Prove that S contains a prime number. (Note" Two positive integers m, n are coprime if their only common factor is 1.)